

Thermal entanglement and teleportation in a dipolar interacting system

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Quantum teleportation, which depends on entangled states, is a fascinating subject and an important branch of quantum information processing. The present work reports the use of a dipolar spin thermal system as a quantum channel to perform quantum teleportation. Non-locality, tested by violation of Bell's inequality and thermal entanglement, measured by negativity, unquestionably show that even entangled states that do not violate Bell's inequality can be useful for teleportation.

I. INTRODUCTION

Quantum mechanics is characterized by its counter-intuitive concepts as, for example, quantum entanglement, whose importance in modern physics has stimulated intensive research of several quantum systems [1, 2]. Entanglement theory treats the quantum correlations among states of a system which may lead to observation of non-local phenomena like the violation of Bell's inequality [3]. Although its great importance, there is not a general operational criterion necessary and sufficient to determine whether a state of arbitrary purity and dimension is entangled or not [1, 2]. However, in a some bipartite systems, like those described in $2 \otimes 2$ and $2 \otimes 3$ Hilbert spaces, several measures, such as negativity, are available for quantifying entanglement [4]. In terms of quantum information purposes, quantum entanglement is an important resource. First, entanglement of a bipartite system represents a fundamental requirement to allow quantum teleportation, which is an information protocol for transmitting an unknown state from one place to another without a physical transmission channel. Such protocol can be seen as a corner stone of quantum information processing because it was the first one showing the usefulness of quantum entanglement [5]. On the other hand, thermal entanglement, i.e., the entanglement of quantum systems at finite temperature, is one of the links between quantum information and condensed matter areas [2], and consequently, it has been extensively studied by both, theoretical and experimental physicists [6–14].

In this work we report the effects of a dipolar interaction between two spins on their degree of entanglement and non-locality. Also, considering such model a quantum communication channel, we analyse the effects of such interaction over its communication capacity through the teleportation fidelity. Such interaction arises due to the influence of the magnetic field created by one magnetic moment on the site of another magnetic moment [15]. We begin with the model of dipolar interaction and show that, for the case of two coupled spins 1/2, whatever is the ground state, we have the presence of entanglement. We quantify the amount of entanglement by using negativity and verify that our model presents some degree of entanglement at a given coupling parameters Δ and ϵ . In addition, we show how the magnetic anisotropies can influence the process of teleportation, which is based on the degree of entanglement of the quantum states involved in the process.

We calculate the averaged teleportation fidelity and verify that this quantity has a similar behavior of negativity and violation of Bell's inequality. Such process successfully occurs without need of nonlocality of quantum states [16–20].

II. THE MODEL

The dipolar interaction arises from the magnetic field created by a magnetic moment of a spin $\vec{\mu} = -\mu_B g \vec{S}$ [15], where μ_B is the Bohr's magneton and g the giromagnetic factor, on the site of another spin and it is represented by the Hamiltonian

$$H = -\frac{1}{3} \vec{S}_1^T \cdot \vec{T} \cdot \vec{S}_2, \quad (1)$$

where $\vec{T} = \text{diag}(\Delta - 3\epsilon, \Delta + 3\epsilon, -2\Delta)$ is a diagonal tensor, $\vec{S}_j = \{S_j^x, S_j^y, S_j^z\}$ is the spin operator, Δ and ϵ are the dipolar coupling constants between the spins. In addition, $\Delta < 0$ indicates that spin is on $x - y$ plane, while $\Delta > 0$ indicates that spin is on z -axis.

That hamiltonian can describe a pair of spin 1/2 particles and can be written in a matrix form

$$H = \frac{1}{6} \begin{pmatrix} \Delta & 0 & 0 & 3\epsilon \\ 0 & -\Delta & -\Delta & 0 \\ 0 & -\Delta & -\Delta & 0 \\ 3\epsilon & 0 & 0 & \Delta \end{pmatrix}, \quad (2)$$

with the respective eigenvalues and eigenvector given by

$$\mathcal{E}_{\Psi^-} = 0, \quad \mathcal{E}_{\Psi^+} = -\Delta/3, \quad \mathcal{E}_{\Phi^{\pm}} = (\Delta \mp 3\epsilon)/6. \quad (3)$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad |\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad (4)$$

Note that the eigenvectors are the four Bell states which are well known bipartite entangled pure states. This hamiltonian can be written through the spectral decomposition in terms of its states, i.e., $H = \sum_{\alpha} \mathcal{E}_{\alpha} |\alpha\rangle\langle\alpha|$, where $\alpha \in \{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}$.

Let's consider the system at thermal equilibrium described by the canonical ensemble $\rho = \mathcal{Z}^{-1} e^{-H/k_B T}$, where H is the hamiltonian of the system, k_B the Boltzmann's constant and

$\mathcal{Z} = \text{Tr}(e^{-H/k_B T})$ the partition function. Since ρ is a thermal operator, the entanglement on this state is called thermal entanglement [22–24]. From using Eq. (2), we obtain the density operator,

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23} & \rho_{22} & 0 \\ \rho_{14} & 0 & 0 & \rho_{11} \end{pmatrix}, \quad (5)$$

where

$$\begin{aligned} \rho_{11} &= \frac{1}{\mathcal{Z}} e^{-\beta\Delta/6} \cosh\left(\frac{\beta\epsilon}{2}\right), \\ \rho_{22} &= \frac{1}{\mathcal{Z}} e^{\beta\Delta/6} \cosh\left(\frac{\beta\Delta}{6}\right), \\ \rho_{23} &= \frac{1}{\mathcal{Z}} e^{\beta\Delta/6} \sinh\left(\frac{\beta\Delta}{6}\right), \\ \rho_{14} &= -\frac{1}{\mathcal{Z}} e^{-\beta\Delta/6} \sinh\left(\frac{\beta\epsilon}{2}\right) \end{aligned} \quad (6)$$

and

$$\mathcal{Z} = 2e^{\beta\Delta/6} \cosh\left(\frac{\beta\Delta}{6}\right) + 2e^{-\beta\Delta/6} \cosh\left(\frac{\beta\epsilon}{2}\right). \quad (7)$$

The parameters Δ and ϵ can be experimentally determined since they are within the partition function \mathcal{Z} and, consequently, within the thermodynamical quantities like magnetic susceptibility and heat capacity.

The density operator can also be written in terms of Bell states as $\rho = \sum_\alpha p_\alpha |\alpha\rangle\langle\alpha|$, where $p_\alpha = \mathcal{Z}^{-1} e^{-\mathcal{E}_\alpha/k_B T}$ is the Boltzmann weight and $\alpha \in \{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}$. Note that the density matrix of the system is expressed in terms of the four Bell states, which are not possible to be written as a convex sum of the original spin states. Thus, the system will always present some degree of entanglement for any of them as the ground state, except when two or more of these states present the same occupation probability, in that case the ground state will be a mixture that produce a separable state.

Generally speaking, the density operator can be written in the Fano form [25]

$$\rho = \frac{1}{4} \left[\mathbb{I} \otimes \mathbb{I} + \vec{r} \cdot \vec{\sigma} \otimes \mathbb{I} + \mathbb{I} \otimes \vec{s} \cdot \vec{\sigma} + \sum_{i,j} c_{ij} \sigma_i \otimes \sigma_j \right], \quad (8)$$

where \mathbb{I} is the 2×2 identity operator, σ_i are the Pauli matrices, $r_j = \langle \sigma_j \otimes \mathbb{I} \rangle$, $s_j = \langle \mathbb{I} \otimes \sigma_j \rangle$, and $c_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$ are spin-spin correlation functions. For our case Eq. (5), $r_j = 0$, $s_j = 0$, $c_{ij} = \delta_{ij} c_i$ and then

$$\rho = \frac{1}{4} \left(\mathbb{I} \otimes \mathbb{I} + \sum_i c_i \sigma_i \otimes \sigma_i \right), \quad (9)$$

where

$$\begin{aligned} c_1 &= \frac{2}{\mathcal{Z}} \left[e^{\beta\Delta/6} \sinh\left(\frac{\beta\Delta}{6}\right) - e^{-\beta\Delta/6} \sinh\left(\frac{\beta\epsilon}{2}\right) \right], \\ c_2 &= \frac{2}{\mathcal{Z}} \left[e^{\beta\Delta/6} \sinh\left(\frac{\beta\Delta}{6}\right) + e^{-\beta\Delta/6} \sinh\left(\frac{\beta\epsilon}{2}\right) \right], \\ c_3 &= \frac{2}{\mathcal{Z}} \left[-e^{\beta\Delta/6} \cosh\left(\frac{\beta\Delta}{6}\right) + e^{-\beta\Delta/6} \cosh\left(\frac{\beta\epsilon}{2}\right) \right]. \end{aligned} \quad (10)$$

The above density operator is named Bell-diagonal state and the coefficients c_i compose a diagonal correlation matrix C . This general form of the density matrix is extremely useful to explore the role of correlations on the system as will be seen in the next section.

III. ENTANGLEMENT AND NONLOCALITY

A. Nonlocality

Bell's inequality, which imposes an upper limit on the correlation between measurements made on observables of separable qubits, is used here to detect non-locality in our system of dipolar interaction. Such inequality states that, in the absence of non-local effects, the correlation between measurements made on two qubits should be limited by ± 2 . However, quantum mechanics imposes limits of $\pm 2\sqrt{2}$ on the same quantities for pure entangled states [3]. Consequently, violation of Bell's inequality is directly related to non-local entangled states [26, 27].

Probably, the most known Bell's inequality is that of Clauser, Horne, Shimony, and Holt (CHSH) [28], which can be tested experimentally [29]. The Bell operator associated with the CHSH quantum inequality is

$$B_{CHSH} = \vec{a} \cdot \vec{\sigma} \otimes (\vec{b} + \vec{b}') \cdot \vec{\sigma} + \vec{a}' \cdot \vec{\sigma} \otimes (\vec{b} - \vec{b}') \cdot \vec{\sigma}, \quad (11)$$

where \vec{a} , \vec{a}' , \vec{b} , and \vec{b}' are unit vectors in \mathbb{R}^3 , and the CHSH inequality is therefore

$$|\langle B_{CHSH} \rangle| \leq 2. \quad (12)$$

If this inequality is violated, the system is non-locally entangled. However, $\langle B_{CHSH} \rangle$ depends on the chosen directions. Therefore, the CHSH inequality can be intentionally violated by choosing the directions that maximize $\langle B_{CHSH} \rangle$. This procedure is defined as

$$\mathcal{B} = \max |\langle B_{CHSH} \rangle|. \quad (13)$$

To determine \mathcal{B} , Horodecki and coworkers [25] proposed a necessary and sufficient condition for a mixed state of two spins 1/2 in the Bell-diagonal form to violate the CHSH inequality. One first define the matrix $U = C^T C$, where C is the correlation matrix and C^T is its transpose. The quantity $M(\rho) = \max_{i < j} (u_i + u_j)$ is defined [25], where u_i are the eigenvalues of the matrix U and the Eq. (13) can be written as $\mathcal{B}(\rho) = 2\sqrt{M(\rho)}$.

It is straightforward to verify that for the present case the matrix is $U = \text{diag}(c_1^2, c_2^2, c_3^2)$. From the eigenvalues of U we obtained M for our system of dipolar interaction and thus the analytical expression for \mathcal{B} is

$$\mathcal{B}(\rho) = 2 \sqrt{\max\{c_1^2 + c_2^2, c_1^2 + c_3^2, c_2^2 + c_3^2\}}. \quad (14)$$

It is worth remember that any violation of the inequality $\mathcal{B}(\rho) \leq 2$ implies non-locality. In Fig. 1(a) we can see $\mathcal{B}(\rho)$

as function of $\Delta/k_B T$ and $\epsilon/k_B T$. The red line is the boundary given by $\mathcal{B}(\rho) = 2$ and thus the states inside the contour defined by the red line are the ones that do not violate the Bell's inequality test, and therefore are local quantum states. It is easy to see that as higher is the ratio $\Delta/k_B T$ and $\epsilon/k_B T$, more states violate Bell's inequality.

B. Entanglement

In general, partial transposition does not retain the positivity required from any density operator, unless, as proven by Horodecki *et al* [30], the state is separable. The concept of negativity [31, 32] is derived from the Peres-Horodecki separability criterion [30, 33] which states that for systems with Hilbert space dimension $2 \otimes 2$ and $2 \otimes 3$, there is entanglement if, and only if, its partial transpose is not positive definite. Thus, the negativity is defined as [32, 34]

$$\mathcal{N}(\rho) = \frac{\|\rho^{T_A}\|_1 - 1}{2}, \quad (15)$$

where $\|A\|_1 = \text{Tr} \sqrt{A^\dagger A}$ is the trace norm, and ρ^{T_A} is the partial transposition of the density operator ρ whose elements are $\langle \alpha\beta | \rho^{T_A} | \gamma\delta \rangle = \langle \gamma\beta | \rho | \alpha\delta \rangle$. As can be proved, negativity is convex and does not increase under local operations and classical communication (LOCC), i.e., it is an entanglement monotone [32]. Moreover, negativity is normalized such that $0 \leq \mathcal{N} \leq 1$, where $\mathcal{N} = 0$ denotes completely separable and $\mathcal{N} = 1$ denotes maximally entangled states [32]. For the present model, Eq. (5), the partial transposition gives

$$\rho^{T_A} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{23} \\ 0 & \rho_{22} & \rho_{14} & 0 \\ 0 & \rho_{14} & \rho_{22} & 0 \\ \rho_{23} & 0 & 0 & \rho_{11} \end{pmatrix}, \quad (16)$$

and so the negativity reads as

$$\mathcal{N}(\rho) = \frac{1}{4} (|c_1 + c_3| + |1 + c_2| + |c_1 - c_3| + |1 - c_2| - 1). \quad (17)$$

Fig. 1(b) shows $\mathcal{N}(\rho)$ as function of $\Delta/k_B T$ and $\epsilon/k_B T$. The red line is the boundary given by $\mathcal{N}(\rho) = 0$ and thus the states inside the contour defined by the red line are the non entangled ones. Again, as higher is the ratio $\Delta/k_B T$ and $\epsilon/k_B T$, more entanglement is present in the system. Comparing the figures for $\mathcal{N}(\rho)$ and $\mathcal{B}(\rho)$ we see that for our system, although both present the same behaviour, not all entangled states are non local in the sense of violating Bell's inequality. Thus, one question may be raised: Considering this system in thermal equilibrium a channel that can promote teleportation, what would be its source for quantum communication capability? Next, section we will discuss details about that question.

IV. TELEPORTATION

The basic idea of quantum teleportation is to transfer a unknown quantum state of a particle to a separated one without a physical communication channel. The seminal proposal

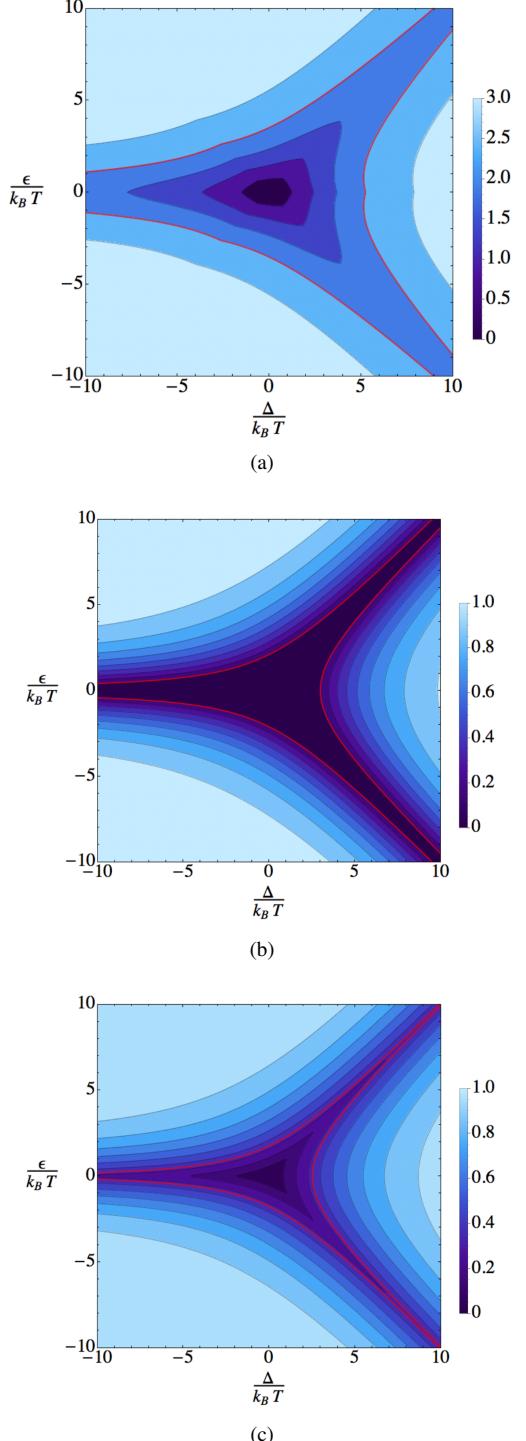


FIG. 1. (Color on line) Bell's inequality violation test $\mathcal{B}(\rho)$ (Fig. 1(a)) and Negativity $\mathcal{N}(\rho)$ (Fig. 1(b)) as function of $\Delta/k_B T$ and $\epsilon/k_B T$. The violation of Bell's inequality occurs in regions where $\mathcal{B} > 2$ and, thus, in this region there is a non local thermal state. Maximum entanglement occurs for values in which $\epsilon \rightarrow 0$ and $\Delta \gg k_B T$ or $\Delta < 0$ and $|\epsilon| \gg k_B T$. Fidelity of teleportation \mathcal{F} (Fig. 1(c)) as function of $\Delta/k_B T$ and $\epsilon/k_B T$. As expected region for \mathcal{F} has the same behaviour of figures of negativity and Bell's inequality violation test. The red lines are the boundary given by $\mathcal{B}(\rho) = 2$, $\mathcal{N}(\rho) = 0$, and $\mathcal{F}_{\min} = 2/3$, the last one being the minimal value to successfully occur the quantum teleportation process.

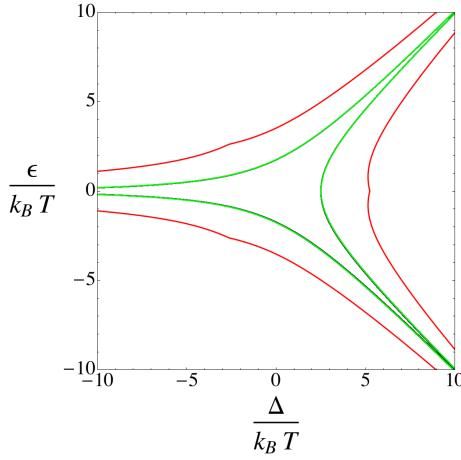


FIG. 2. (Color on line) Superposition of the red lines in figures 1 in terms of $\Delta/k_B T$ and $\epsilon/k_B T$. The red line is to refer Bell ($\mathcal{B}(\rho) = 2$), the black line is to refer Negativity ($\mathcal{N}(\rho) = 0$), and the green line is to refer Fidelity ($\mathcal{F}(\rho) = 2/3$). In this figure is emphasized that all the entangled states are useful for teleportation.

of Bennet and co-workers presents a teleportation protocol based on Bell measurements, which are projective measurements over one of the four states of the Bell basis (see Eq. (4)), and Pauli matrices rotations. The protocol can be described as follows [35]. Consider a pair of particles prepared in an entangled pure state like the singlet state. One of them remains in laboratory A with Alice and the other is sent to laboratory B to Bob, which is far apart creating a noiseless quantum channel. Alice wants to communicate to Bob the unknown state of a third particle. She performs a Bell measurement over the two particles in her lab and then call Bob to tell him which measurement she did. Thus, using this information Bob determines which unitary rotation he must perform to obtain the original unknown state of the third particle. After the appearance of this protocol many others were proposed and experimentally tested, even considering noisy quantum channels [35] as for example thermalized two-qubit Heisenberg models [36–39].

Thus, considering the standard teleportation protocol using a noisy quantum channel as proposed by Bowen and Bose [35], let's suppose that the initial unknown state to be teleported is

$$|\psi(\theta, \phi)\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle \quad (18)$$

whose corresponding density operator is

$$\rho_{in} = \frac{1}{2} (\mathbb{I} + \hat{n} \cdot \vec{\sigma}) , \quad (19)$$

where \mathbb{I} is the identity matrix, $\hat{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. Such density operator represents the general state of a single qubit in the Bloch sphere.

The output state of this protocol is given by [35]

$$\rho_{out} = \sum_{\mu=0}^3 \text{Tr}(\mathcal{K}_\mu \rho) \sigma_\mu \rho_{in} \sigma_\mu , \quad (20)$$

where $\mathcal{K}_\mu = \sigma_\mu \mathcal{K}_0 \sigma_\mu$, and $\sigma_0 = \mathbb{I}$, $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$ and $\sigma_3 = \sigma_z$, and we choose, without loss of generality, $\mathcal{K}_0 = |\Psi^+\rangle\langle\Psi^+|$ based on the possible ground states of the system in Eqs. (4). For the present case, we consider the density matrix ρ given in Eq. (9) representing the noisy teleportation channel where the source of noise is the temperature.

After some simple algebra it is possible to conclude that

$$\sigma_\mu \rho_{in} \sigma_\mu = \frac{1}{2} \mathbb{I} + \sum_{j=1}^3 \left(\delta_{j\mu} - \frac{1}{2} \right) n_j \sigma_j . \quad (21)$$

Thus, the final density matrix of the output state has the form

$$\begin{aligned} \rho_{out} = & \frac{1}{2} \mathbb{I} + \frac{1}{2} (2p_{\Psi^+} + 2p_{\Phi^+} - 1) \sin \theta \cos \phi \sigma_1 \\ & + \frac{1}{2} (2p_{\Psi^+} + 2p_{\Phi^-} - 1) \sin \theta \sin \phi \sigma_2 \\ & + \frac{1}{2} (2p_{\Psi^+} + 2p_{\Psi^-} - 1) \cos \theta \sigma_3 . \end{aligned} \quad (22)$$

where $p_{\Phi^\pm} = \mathcal{Z}^{-1} e^{-\beta E_{\Phi^\pm}}$ and $p_{\Psi^\pm} = \mathcal{Z}^{-1} e^{-\beta E_{\Psi^\pm}}$ are the Boltzmann weights, i.e., the occupation probability of each state. With the initial and final states, we may calculate the teleportation fidelity [20, 40, 41] $f_{\theta, \phi}(\rho_{in}, \rho_{out}) = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$. Since the parameters θ and ϕ are unknown, we calculate the average of $f_{\theta, \phi}(\rho_{in}, \rho_{out})$ over the Bloch sphere. Thus,

$$F = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta f_{\theta, \phi}(\rho_{in}, \rho_{out}) . \quad (23)$$

Evaluating $f_{\theta, \phi}(\rho_{in}, \rho_{out})$ and replacing in Eq. (23), we then obtain

$$F_{|\Psi^+\rangle} = \frac{1}{3} (1 + 2p_{\Psi^+}) , \quad (24)$$

where the index $|\Psi^+\rangle$ is just to refer our initial choice of \mathcal{K}_0 . Similarly, we can do the same procedure for the others possible choices of \mathcal{K}_0 considering our system and obtain

$$F_{|\alpha\rangle} = \frac{1}{3} (1 + 2p_\alpha) , \quad (25)$$

where $\alpha \in \{\Psi^+, \Psi^-, \Phi^+, \Phi^-\}$. Note that $1/3 < F_{|\alpha\rangle} < 1$. The set of equations (25) shows that the teleportation fidelity is directly related to occupation probability of the ground state considered.

We have seen early that for different ranges of Δ and ϵ the ground state of the system changes. Thus, considering this fact, the expression for teleportation fidelity will be F_{Φ^+} , F_{Φ^-} , and F_{Ψ^+} depending on the highest value of fidelity, i.e., the value of fidelity related to the highest occupation probability of each eigenstate. In Fig. 1(c) we have the function

$$\mathcal{F} = \max (F_{|\Phi^+\rangle}, F_{|\Phi^-\rangle}, F_{|\Psi^+\rangle}) . \quad (26)$$

in terms of $\Delta/k_B T$ and $\epsilon/k_B T$. The teleportation fidelity is maximum in the same region that indicates maximum entanglement ($\mathcal{N} = 1$), so that the teleporting process successfully

occurs. However, it is also necessary to know which is the minimal value of fidelity \mathcal{F} to successfully occur quantum teleportation. In Refs. [42, 43] the authors show that the minimal fidelity \mathcal{F} is inversely proportional to dimension d of Hilbert subspace of a bipartite system ($\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathbb{C}^d \otimes \mathbb{C}^d$), i.e.,

$$\mathcal{F}_{min} = \frac{2}{1+d}. \quad (27)$$

For our system $d = 2$ and we have that $\mathcal{F}_{min} = 2/3$; in accordance with that result from Ref. [44]. This boundary is the red line in Fig. 1(c). It means that below this value, the teleporting process ceases. As expected \mathcal{F} has the same behavior of figures of negativity and Bell's inequality which means that the non possibility to have entangled ground state is, indeed, important. This indicates that entanglement and not only non-locality of the ground state is an important condition to perform quantum teleportation. The Fig. 2 shows simultaneously the three lines (red lines in Fig. 1) previously discussed. It is evident in this figure that exists a region (between the green and the red line in Fig. 2) where the states are local but entangled and still the teleportation process successfully occurs.

V. DISCUSSIONS

In Sec. III we study the influence of the dipolar interaction on the thermal entanglement present in a system of two spins-1/2. Thus, our aim is to know how the magnetic anisotropies, represented by the coupling parameters Δ and ϵ affect the entanglement, the non-locality and, consequently the teleportation process. In order to observe the nonlocal correlations we use a measurement based on Bell's inequality called Bell measurement. This quantity is directly related to nonlocal entangled states. In addition, the thermal entanglement of the system's state was quantified using the negativity. Both quantities were presented in Fig. 1 as a function of $\Delta/k_B T$ and $\epsilon/k_B T$. In Fig. 1(a) we have regions where $\mathcal{B}(\rho) > 2$ which means a violation of Bell's inequality and, consequently, the presence of non local states. Maximum entanglement occurs for values in which $\epsilon \rightarrow 0$ and $\Delta \gg k_B T$ or $\Delta < 0$ and $|\epsilon| \gg k_B T$ which corresponds to lightest blue region in Figs. 1(a) and 1(b). However, Fig. 1(b) shows a region, the darkest blue, where negativity is zero and the state of the system is separable. The presence or absence of entanglement is related to the ground state of the system which can change as function of ϵ and Δ . The change of the ground state can be understood through the variation of the Boltzmann's weights which provide the occupation of the energy levels. Thus, we define the function

$$P = \max\{p_{\Phi^+}, p_{\Phi^-}, p_{\Psi^+}, p_{\Psi^-}\}, \quad (28)$$

i.e., we take the highest value among Boltzmann's weights. Fig. 3 presents Eq. (28) in terms of $\Delta/k_B T$ and $\epsilon/k_B T$ and has the same behavior of negativity \mathcal{N} and Bell's inequality violation test \mathcal{B} . In addition, in this figure we identify each region with the dominant Boltzmann's weight. Note that the

weight p_{Ψ^-} does not appear in the figure due to its lower value compared to the others. The coupling parameters Δ and ϵ determine which of the four Bell eigenstates is the ground state of the system. It is easy to see by using the set of Eqs. (3) and the Boltzmann weights p_a that in the limit $(\Delta, \epsilon) \ll k_B T$ and $\Delta < 0$ the ground state is degenerated, this produce a separable mixture of bell states. For the limits $\Delta \gg k_B T$ the occupation probability is $p_{\Psi^+} = 1$ while for $\epsilon \gg k_B T$, $p_{\Phi^-} = 1$ and for $-\epsilon \gg k_B T$, $p_{\Phi^+} = 1$. In the last three situations we have a pure and maximally entangled ground state. From the above results we study the possibility of using such system as a quantum channel in order to perform quantum teleportation. The fidelity of teleportation as described in Sec. IV allow us to know if our system constitute a good channel. As we have seen in that section, depending on the choice of \mathcal{K}_0 we have different expressions for the fidelity, each one is related to its respective Boltzmann's weight.

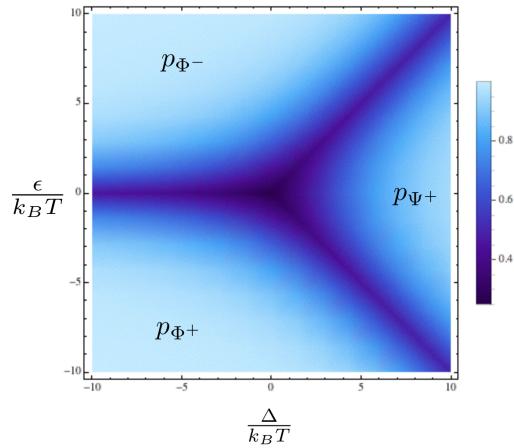


FIG. 3. (Color on line) Region for function P , from Eq. (28), in terms of $\Delta/k_B T$ and $\epsilon/k_B T$. Each region is identified with the dominant Boltzmann's weight.

VI. CONCLUSIONS

To conclude, we demonstrate the effects of the coupling parameters of dipolar interaction between two spins 1/2 particles on the entanglement and non locality of thermal states. This effect is also reflected in the quantum teleportation process. We began by describing the model and showed that the eigenstates of the system may be written in terms of Bell states. We then discussed negativity and the violation of Bell's inequality for this system in order to check the amount of entanglement and the nonlocal correlations present. We showed that the system presents maximally entangled states in the region where $\epsilon \rightarrow 0$ and $\Delta \gg k_B T$ or $\Delta < 0$ and $|\epsilon| \gg k_B T$ ($\Delta, \epsilon \gg k_B T$, according to negativity).

We then proceeded by providing an application for such model in a quantum teleportation process. We evaluated the fidelity of teleportation \mathcal{F} , which depends on the probability of occupation of each Bell state, and showed that this quantity

is maximum when entanglement is maximum and the teleportation process ceases when there is no entanglement as expected. For this system, the minimal value of \mathcal{F} to successfully occurs quantum teleportation is $2/3$. As a remarkable result we found that, even without violation of the Bell inequality, the teleportation successfully occurs as shows the Fig. 2. Thus, for the present system, all the entangled states are useful for teleportation.

Finally, this study provides relevant insights into how the parameters of dipolar interaction coupling between two spins $1/2$ particles affect entanglement, violation of Bell's inequality, and teleportation fidelity. Thus, we advocate that this

model of dipolar interaction can be used as a good quantum channel to promote quantum communication.

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